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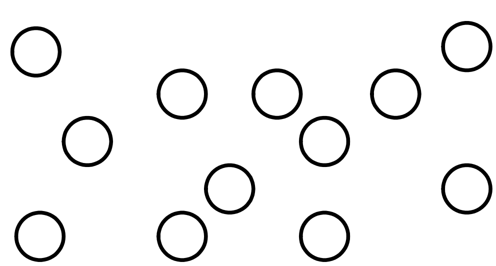
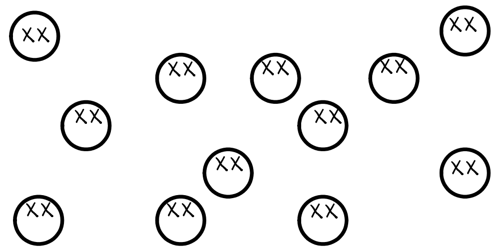
Irv was talking about solving a word problem and told us of Sherry's clever non-algebraic solution. I want to make two points in this post. One is that the usual casual conversion of words to algebra hides real issues and two is that algebraic steps have their own story that can be expressed in words. Mathematics was added to the curriculum of educated people because of its power to simplify real world problems. Over time, math education lost this connection in a welter of symbolic steps and rote applications.

The problem as originally presented was: You have goats and hens with a total of 12 noses and 34 feet. How many goats and how many hens are there? "Wait! Do hens have noses? I'm lost." These types of issues hang up some students and make others distain the whole enterprise. So let's state the problem more carefully.

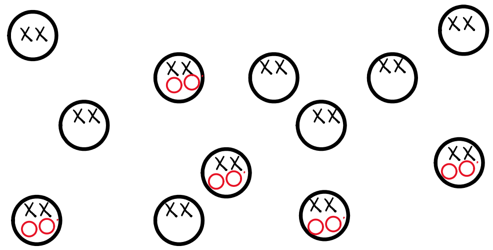
> We are in a dog park. The dogs and their human handlers have 12 noses and 34 feet between them. How many dogs and how many humans are in the dog park?
>
> Solution with comments:
>
> > Let d stand for the number of dog noses in the park and h stand for the number of human noses in the dog park.
> >
> > > (We had other choices for the variables, for instance, they could have represented the number of dogs and the number of humans or even with a bit of contortion the number of dog legs and the number of human legs. In these cases, we would have had to translate using common knowledge, like animals have only one nose so there is a one-to-one correspondence between noses and animals, to get to our answer. As an exercise, try working the problem with these other variable specifications. We can now form some equations.
> >
> > First $d + g = 12$
> >
> > > (Already we are hiding some information. What are the units of the 12 ? The equation is better written:
> > >
> > > $d\ dog\ noses + h\ human\ noses = 12\ noses$)
> >
> > Before we talk about the second equation, let's see Sherry's pictorial method. First make 12 circles representing the humans and dogs with one (unindicated nose) per being.



Then put two x's in each circle to represent the humans' two legs or the dogs' back legs.



Count the x's (24), subtract from 34 to get the remaining front legs of the dogs. (10) Put two red circles each in 5 circles to represent the dog's front legs. So we get only 5 dogs and the rest (7) are humans.



There are five dogs and seven humans in the dog park.

Now return to the algebraic solution. We need the equation (constraint) provided by the leg restriction. This would be,

$4d + 2h = 34$

(Inserting the units will help make sense of this.

$$4\,\frac{legs}{dog\ nose} \cdot d\ dog\ noses + 2\,\frac{legs}{human} \cdot h\ human\ noses = 34\ legs$$

It is often convenient to treat units as algebraic objects since canceling keeps the units balanced. Here we have written two different looking unit descriptors for the same unit, $nose$ and $noses$. My (our?) need to make a grammatical sentence out of the equation lead us down this path.)

A next algebra step might be to multiple the first equation by two.

$2(d + h = 12)$

$2d + 2h = 2 \cdot 12$

$2d + 2h = 24$

(This is a beginning in an attempt to count legs in a different way allocating them like Sherry did among the beings in the park. A fresh notation for legs is necessary.

$$2\,\frac{(back\ dog\ legs)}{dog\ nose} \cdot d\ dog\ noses + 2\,\frac{legs}{human\ nose} \cdot h\ human\ noses$$
$$= 24\ back\ dog\ legs\ and\ human\ legs$$

We can see now why we fight to get a problem into algebra as soon as possible. That lets us ignore all these issues. But remember, if we need to troubleshoot, we may have to dive into unit land.)

We now have the hint for rewriting the second equation.

$2d + 2d + 2h = 34$

(With units, this says

$2\,\frac{(front\ dog\ legs)}{dog\ nose} \cdot d\ dog\ noses + 2\,\frac{(back\ dog\ legs)}{dog\ nose} \cdot d\ dog\ noses + 2\,\frac{legs}{human\ nose}$

$\cdot\, h\ human\ noses = 34\ legs$)

We can replace the last two expressions on the left side of the equation with the right side of the last equation,

$2d + 24 = 34$ or

($2\,\frac{(front\ dog\ legs)}{dog\ nose} \cdot d\ dog\ noses + 24\ back\ dog\ legs\ and\ human\ legs = 34\ legs$)

So,

$2d = 34 - 24$

$2d = 10$

($2\,\frac{(front\ dog\ legs)}{dog\ nose} \cdot d\ dog\ noses = 34\ legs - 24\ back\ dog\ legs\ and\ human\ legs$

$2\,\frac{(front\ dog\ legs)}{dog} \cdot d\ dog\ noses = 10\ front\ dog\ legs$)

Then, divide both sides by two.

$\frac{2d}{2} = \frac{10}{2}$

$d = 5$

($(2\,\frac{(front\ dog\ legs)}{dog\ nose} \cdot d\ dog\ noses)/2\,\frac{(front\ dog\ legs)}{dog\ nose} = 10\ front\ dog\ legs\ )/2\,\frac{(front\ dog\ legs)}{dog\ nose}$

Reduce units to get,

$d\ dog\ noses = 5\ dog\ noses$)

Finally using our original equation,

$d + h = 12$

$5 + h = 12$

$h = 7$

And there are 5 dogs and 7 humans.

($d\ noses + h\ noses = 12\ noses$

$5\ dog\ noses + h\ noses = 12\ noses$

$h = 7\ noses$)

Our original algebra, gives use the answer that there are 5 dogs and 7 humans in the dog park. Using the units as we have, we must first state that since the being noses and the beings in the problem are in one-to-one correspondence, there are 5 dogs and 7 humans in the dog park. (By the way I played golf with a gentleman with half a nose once.)

So, applied algebra can have its own reality based story and care with units helps clarify and check answers.